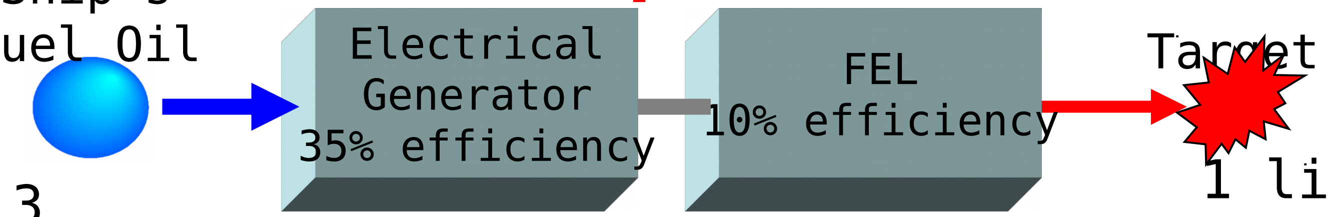
PH4911
“Free Electron Lasers”

Physics Department
Naval Postgraduate School

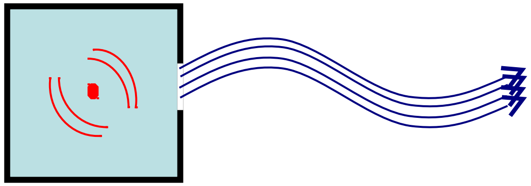
"FREE" Electron Laser Weapon



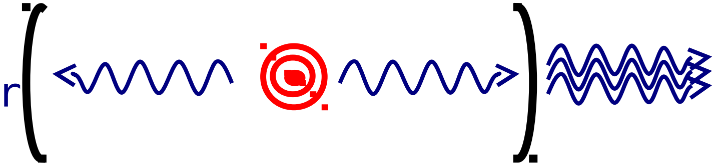
FEL @ 10% efficiency for 5 s ⇒ 50 MJ needed
oil contains 110 MJ / gallon
00 gas-turbine generator has 35% efficiency
s engagement uses a few gallons of fuel at a few
ational cost is important for a Naval weapon system
missile defense engagement costs $ 1,500,000
ANX missile engagement cost $ 3,000
l training with missiles is COSTLY
time cost of FEL maybe very reasonable !!

FEL General History

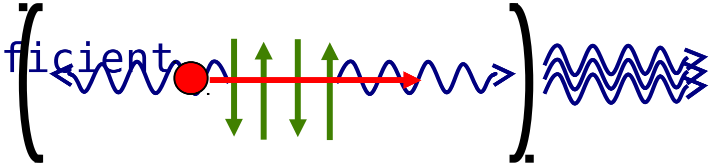
- Microwave Tubes (1930’s)
  Uses beam of free non-relativistic electrons
  + closed microwave cavity
  ⇒ long wavelengths & efficient



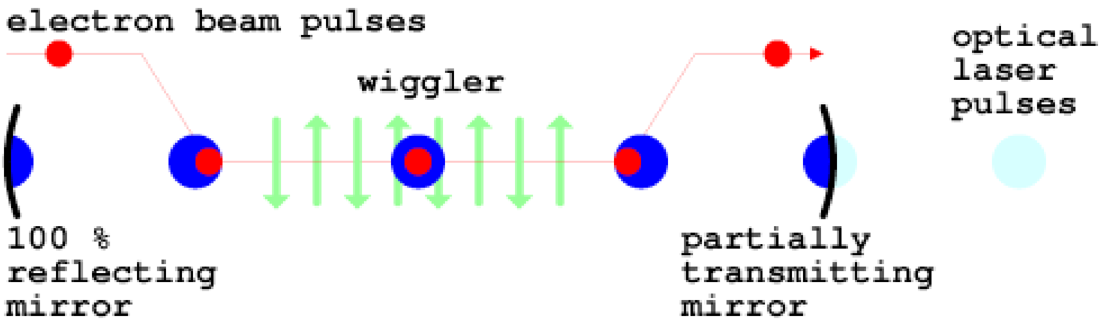
- Atomic and Molecular Lasers (1960’s)
  Uses system of bound electrons
  + open optical resonator
  ⇒ short wavelengths, not tunable or efficient



- Free Electron Laser (Madey 1970’s)
  Uses beam of free relativistic electrons
  + open optical resonator
  ⇒ short wavelengths, tunable, & efficient



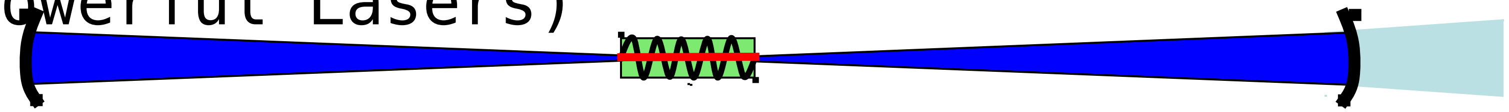
FEL Attributes



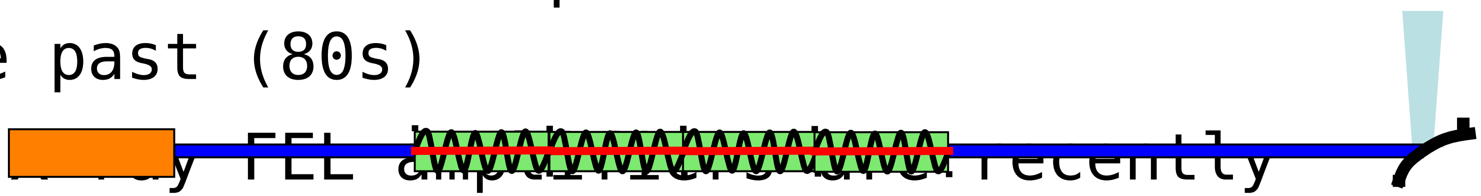
- FELs are continuously tunable: $\lambda = \lambda_o(1+K^2)/2\gamma^2$
- FELs are designable:
  microwaves → infrared → visible → UV → X-Rays
- FELs are powerful: laser medium cannot be damaged
- FELs are efficient: ≈ 10 % (> 60 % for microwave tubes)
- FELs are reliable: systems now run 24 hrs/day for weeks

FEL Oscillators and Amplifiers

- Most FELs have been oscillators
- Jefferson laboratory is an FEL oscillator
- WSMR uses laser oscillators (our most powerful Lasers)

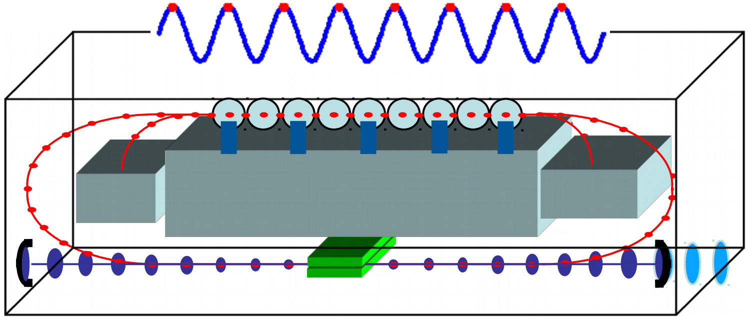


- Several FEL amplifiers have been built in the past (80s)
- X-ray FEL amplifiers are recently developed (late 90s)



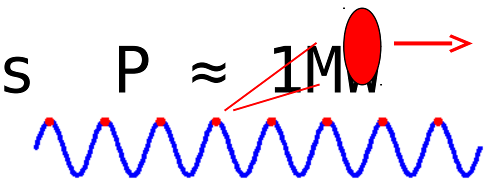
- **BUT** FELs can do ***both*** the amplifier

Amplifier/Oscillator Requirements



- **Laser Wavelength:**
  $E_b \approx 90\text{MeV}$, $\lambda_o \approx 3\text{cm}$, $K \approx 2$
  $\Rightarrow \lambda = \lambda_o(1+K^2)/2\gamma^2 \approx 2\mu\text{m}$

- **Electron Beam Power:** (E_b set by wavelength requirement)
  - $E_b \approx 90$ MeV and $I_{avg} \approx 0.7\text{A} \Rightarrow P_b \approx 70$ MW
  - Extraction $\eta \approx 2\%$ gives $P \approx 1\text{MW}$ optical output power



- **RF accelerator and FEL Gain requires high peak current:**
- RF & pulse repetition frequency: f ≈

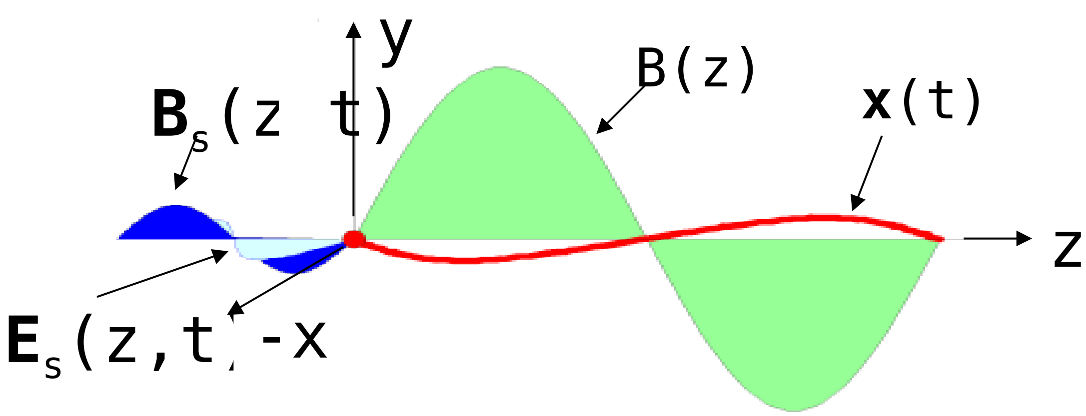
Relativistic Equations: Planar Undulator

- Relativistic Equations of Motion:

$$\frac{d(\gamma\vec{\beta})}{dt} = -\frac{e}{mc}(\vec{E} + \vec{\beta}\times\vec{B}) \qquad \dot{\gamma} = \frac{d\gamma}{dt} = -\frac{e}{mc}\vec{\beta}\cdot\vec{E} \qquad , \quad \gamma^{-2} = 1-\vec{\beta}^2$$

- Undulator Field: **B** = B(0, $\sin k_0 z$, 0) (linear polarization)
- Optical Fields: $\mathbf{E}_s$=E($\cos\psi$,0,0), $\mathbf{B}_s$=E(0,$\cos\psi$,0), $\psi = kz-\omega t+\phi$

  where $k=2\pi/\lambda$, ω=kc optical frequency , ϕ = optical phase



FEL Theory: Microscopic Motion

- Relativistic equations → electron's microscopic motion

Simple Pendulum Equation: $\boxed{\overset{\circ\circ}{\zeta} = \overset{\circ}{\nu} = |a|\cos(\zeta + \phi)}$

- Dimensionless Optical field: $|a| = 4\pi NeKLE/\gamma^2 mc^2$
- Electron phase on λ scale: $\zeta = (k+k_0)z - \omega t \approx 2\pi\Delta z/\lambda$
- Electron phase velocity: $\nu = \overset{\circ}{\zeta} = L[(k+k_0)\beta_z - k] \approx 4\pi N(\gamma - \gamma_R)/\gamma_R$
- Resonate electron energy: $\gamma_R = \sqrt{\lambda_0(1+K^2)/2\lambda}$ $\quad \lambda = \lambda_0(1+K^2)/2\gamma_R{}^2$
- Dimensionless

$\tau = ct/L$

Note: $\overset{\circ}{(\cdots)} = \dfrac{d(\cdots)}{d\tau}$

optical fields $a < \pi$ ⇒ "weak bunching" in "weak fields" ⇒ finite gain

g optical fields have $a > \pi$ ⇒ strong bunching, sa

FEL Pendulum Phase Space

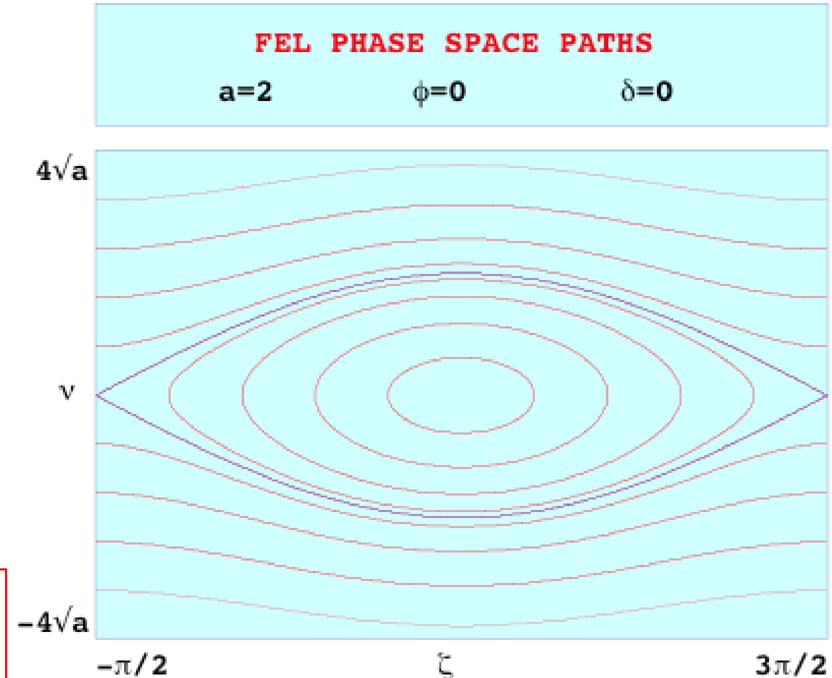
- FEL pendulum equation

$$\overset{\circ\circ}{\zeta} = \frac{d^2\zeta}{d\tau^2} = \overset{\circ}{\nu} = \frac{d\nu}{d\tau} = |a| \cos(\zeta + \phi)$$

- FEL separatrix

$$\nu_s^2(\zeta_s) = 2|a|[1 + \sin(\zeta_s + \phi)]$$

- FEL electrons only evolve for time $\tau = 0 \rightarrow 1$



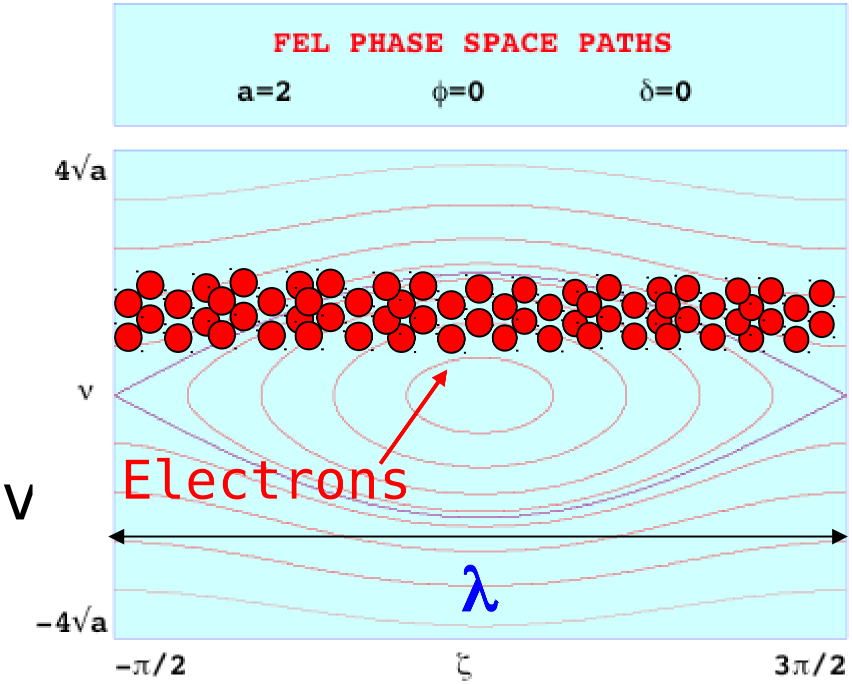
electron beam has many electrons in each wavelengt

10^6 - 10^7 random ζ_0's within in each optical wavele

- ν-axis follows electron energy in phase space
- ζ-axis follows electron microscopic position in phase space

$$\nu_p^2(\zeta_p) = \nu_0^2 + 2|a|[\sin(\zeta_p + \phi) - \sin(\zeta_0 + \phi)]$$

FEL Pendulum Phase Space



- FEL pendulum equation

$$\overset{\circ\circ}{\zeta} = \frac{d^2\zeta}{d\tau^2} = \overset{\circ}{\nu} = \frac{d\nu}{d\tau} = |a|\cos(\zeta + \phi)$$

- FEL separatrix

$$\nu_s^2(\zeta_s) = 2|a|[1 + \sin(\zeta_s + \phi)]$$

- FEL electrons only evolv
  - for time $\tau = 0 \rightarrow 1$
  - $\approx 10^6$ - 10^7 random ζ_0's within in each optical wavelength λ
  - ν-axis follows electron energy in phase space

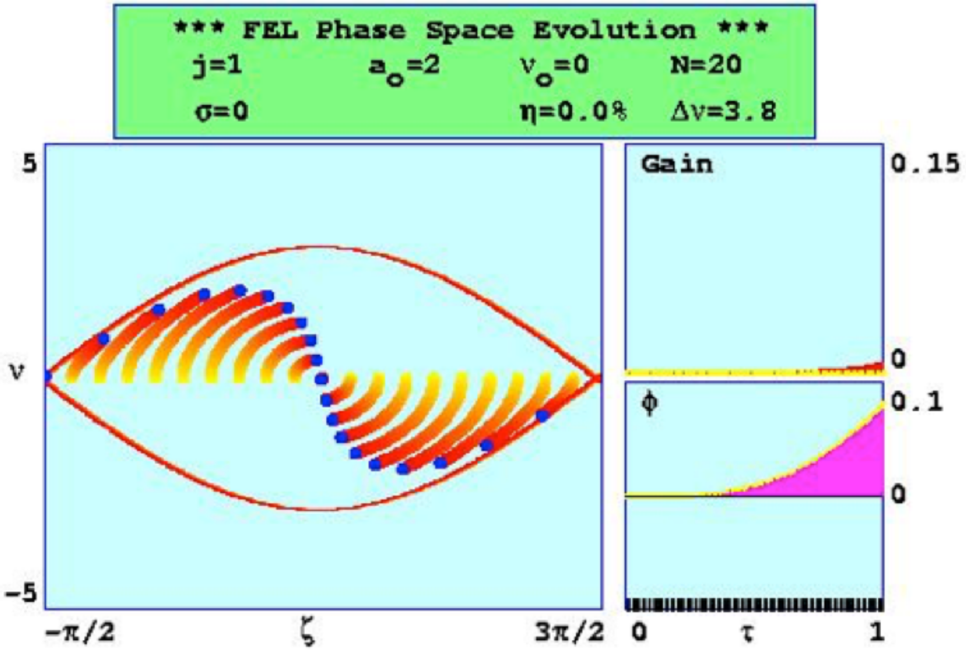
FEL Phase Space Evolution

- Use 20 sample electrons
- Follow all paths in (ζ, ν)
- Gain: $G(\tau) = (|a(\tau)|^2 - a_0^2)/a_0^2$
- Optical phase: $\phi(\tau)$
- Evolution for $\tau = 0 \rightarrow 1$
- Separatrix path given b

$$\nu_s^2(\zeta_s) = 2|a|[1 + \sin(\zeta_s + \phi)]$$

- Separatrix height: 2a
- Initial Conditions:

  Weak fields: $a_0 = 2 < \pi$
- At resonance: $\nu_0 = 0$



$$\overset{\circ\circ}{\zeta} = \frac{d^2\zeta}{d\tau^2} = \overset{\circ}{\nu} = \frac{d\nu}{d\tau} = |a|\cos(\zeta + \phi)$$

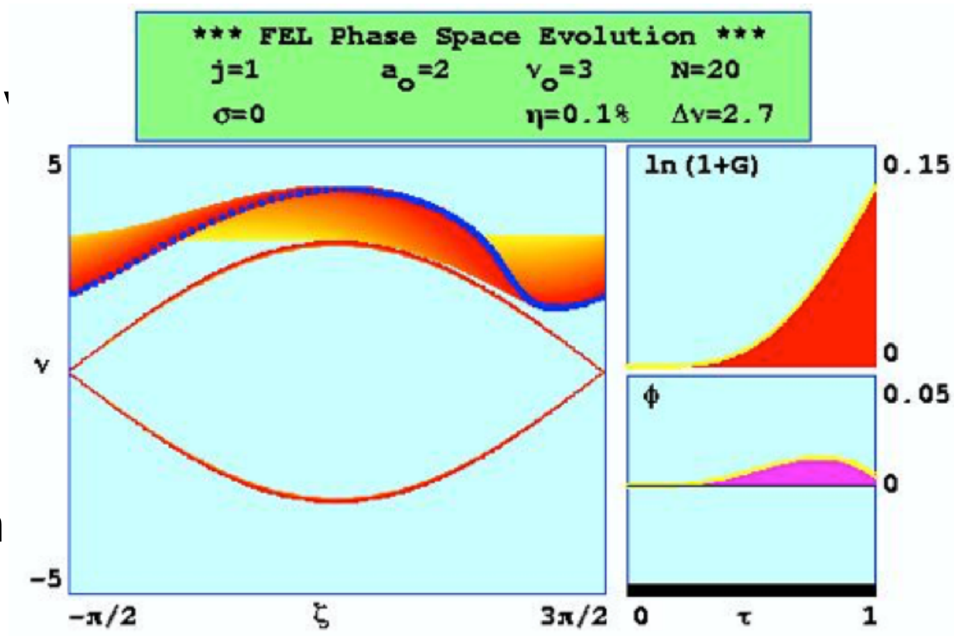
FEL Pendulum Equation

FEL Phase Space Evolution

- 200 sample electrons
- Follow all paths in $(\zeta,$
- Gain:

$$G(\tau) \propto \frac{<\gamma(\tau)>}{E^2} \propto \frac{<\Delta\nu(\tau)>}{a_0^2}$$

- <…> beam average
- $G \propto \rho \propto I$ = beam current
- Separatrix height: $2|a|^{1/2}$
- Initial Conditions:
  Weak fields: $a_0=2 < \pi$
- Off resonance: $\nu_0=3$



$$\overset{\circ\circ}{\zeta} = \frac{d^2\zeta}{d\tau^2} = \overset{\circ}{\nu} = \frac{d\nu}{d\tau} = |a|\cos(\zeta+\phi)$$

FEL Pendulum Equation

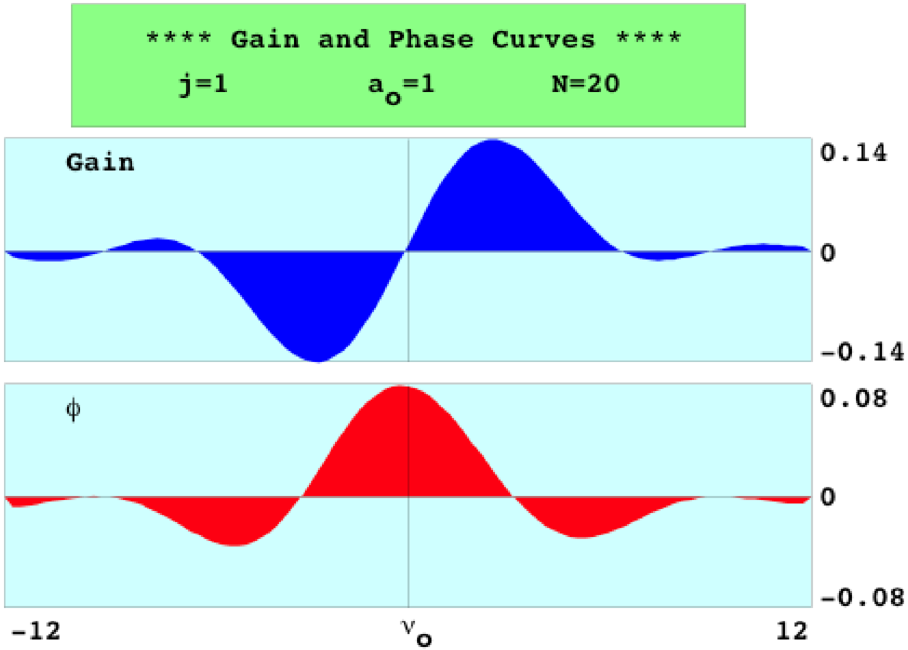
FEL Gain and Phase Spectra

- Evaluate Gain $G(\nu_0)$ at $\tau=1$

$$G = j\frac{[2-2\cos(\nu_0)-\nu_0\sin(\nu_0)]}{\nu_0^3}$$

$$\phi = j\frac{[2\sin(\nu_0)-\nu_0(1+\cos(\nu_0))]}{2\nu_0^3}$$

$$j = \frac{8N(e\pi KL)^2\rho F}{\gamma^3 mc^2}$$



- $G(\nu_0)$ is anti-symmetric in ν_0
- $\phi(\nu_0)$ is symmetric in ν_0

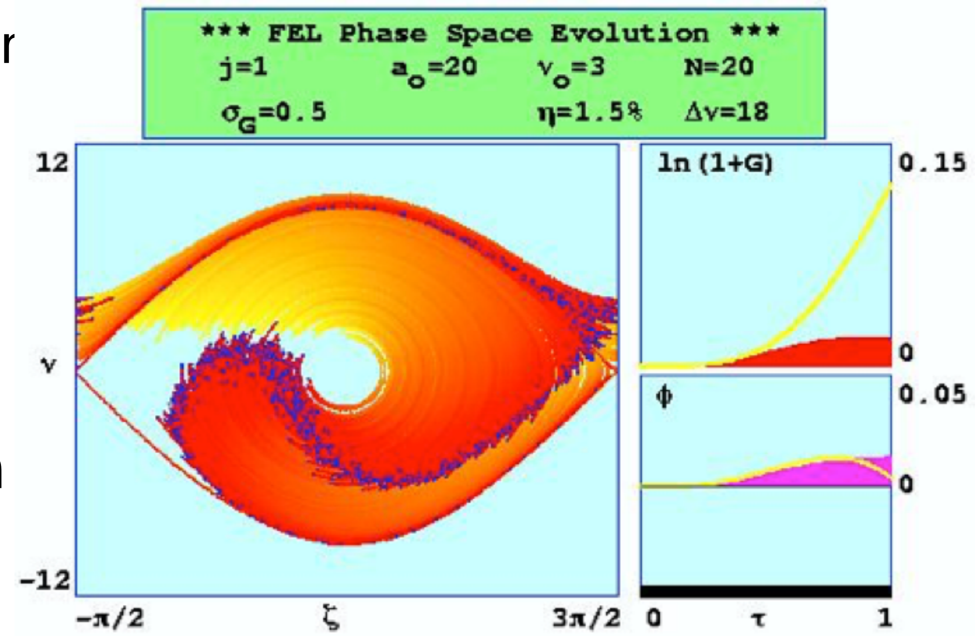
$$\nu_0 = L[(k+k_0)\beta_z(0)-k] \approx 4\pi N\Delta\gamma/\gamma \approx 2\pi N\Delta\lambda/\lambda$$

Strong Field FEL Phase Space Evolution

- Use 1000 sample electron
- Initial Conditions:
  Strong fields: $a_0=20 > \pi$
- Off resonance: $\nu_0=3$
- Electron bunch takes energy back from laser beam
- Gain reduced
  $G \approx 13\% \rightarrow 3\%$
- Separatrix path given by

$$\nu_s^2(\zeta_s) = 2|a|[1+\sin(\zeta_s+\phi)]$$

- Separatrix height: $2|a|^{1/2}$



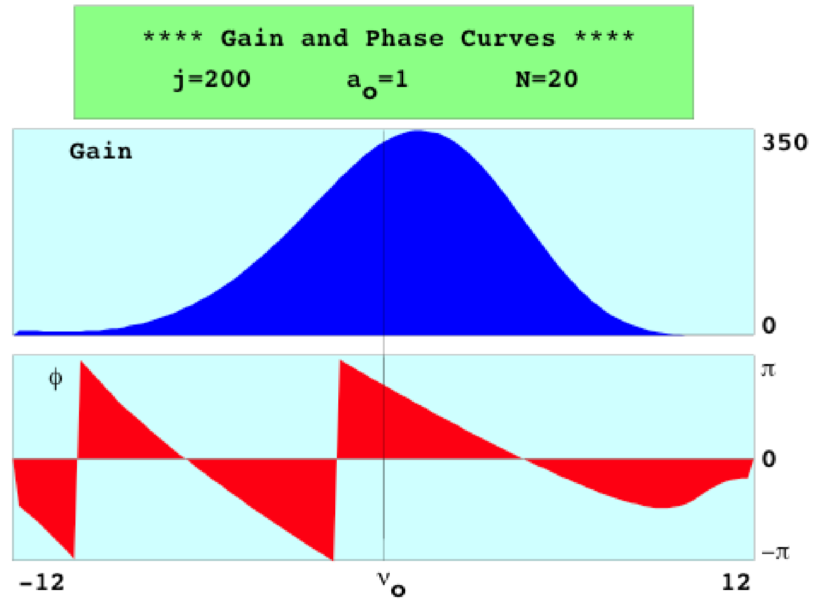
- Spread $\Delta\nu_0=\sigma_G=0.5$ due to electron beam energy spread
- $\sigma_G=0.5< \pi$ is

High Current FEL
Gain and Phase Spectra

- For large current $j \gg \pi$ optical field grows exponentially

$$G(\tau) \approx \exp[(j/2)^{1/3}\sqrt{3}\tau]/9$$

$$\phi(\tau) \approx (j/2)^{1/3}\tau/2$$



- Gain at $\tau=1$:

$$G(\tau) \approx \exp[(j/2)^{1/3}\sqrt{3}]/9 \approx 350 \gg 1$$

- Optical phase significant !!
- $G(\nu_0)$ is $\approx$ symmetric in ν_0

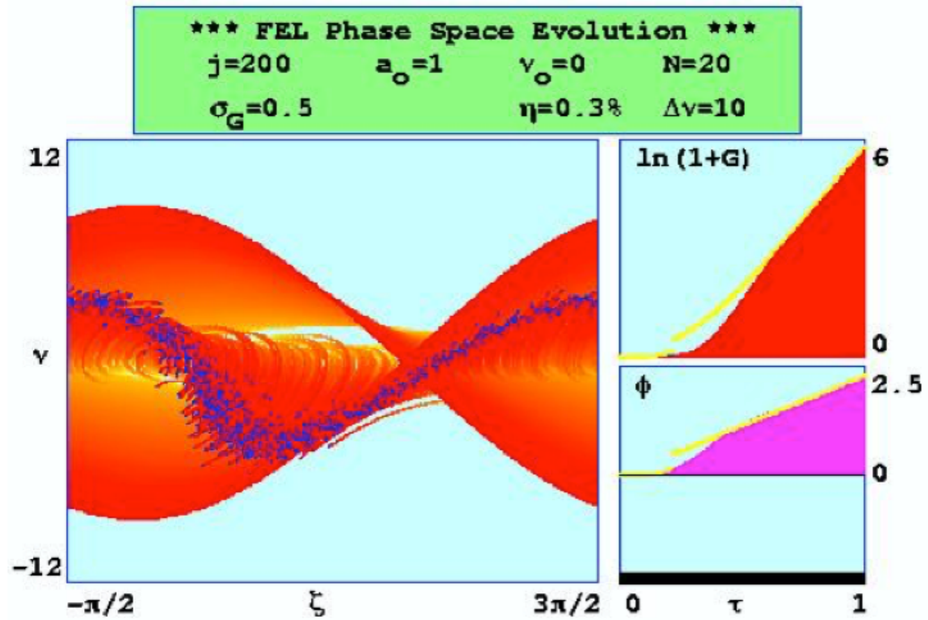
$$j = \frac{8N(e\pi KL)^2\rho F}{\gamma^3 mc^2} \gg \pi$$

- Significant gain at resonance $\nu_0=0$

$$\nu_0 = L[(k+k_0)\beta_z(0) - k] \approx 4\pi N\Delta\gamma/\gamma \approx 2\pi N\Delta\lambda/\lambda$$

High Current FEL Phase Space Evolution

- High Current j=200 >> π
- Follow 1000 electrons
- Gain is exponential in τ
- phase ϕ is linear in τ
- $|a(\tau)|$ & $\phi(\tau)$ self-consistent
- Separatrix grows and shifts with $|a(\tau)|$ & $\phi(\tau)$
- Weak field Gain at $\nu_0=0$
  - 1st - no gain, but ϕ > 0
  - 2nd - bunching at $\zeta \approx \pi/2$
  - 3rd - bunch lowers < ν >



$$\ddot{\zeta} = \dot{\nu} = |a|\cos(\zeta+\phi)$$

$$|\dot{a}| = -j < \cos(\zeta+\phi) >$$

$$\dot{\phi} = j < \sin(\zeta+\phi) > / |a|$$

FEL Pendulum & Wave Equation

Tapered FEL Undulator Designs

Conventional Periodic Undulator: $\delta = 0$

(you already saw these)

Positively Tapered Undulator: $\delta > 0$

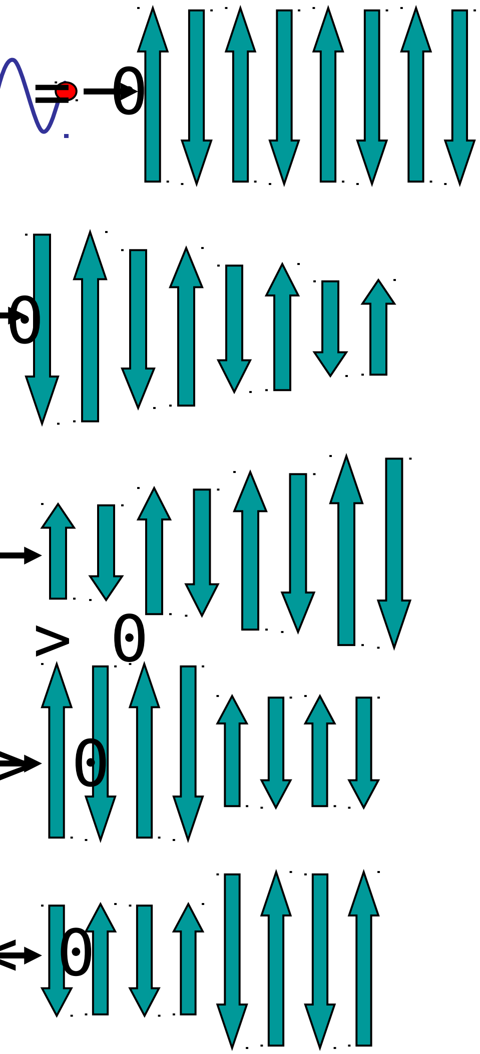
$K\downarrow$ & $\gamma\downarrow$ so $\lambda = \lambda_0(1+K^2)/2\gamma^2$

gatively Tapered Undulator: $\delta < 0$

not intuitive, but works for $\nu_0 > 0$

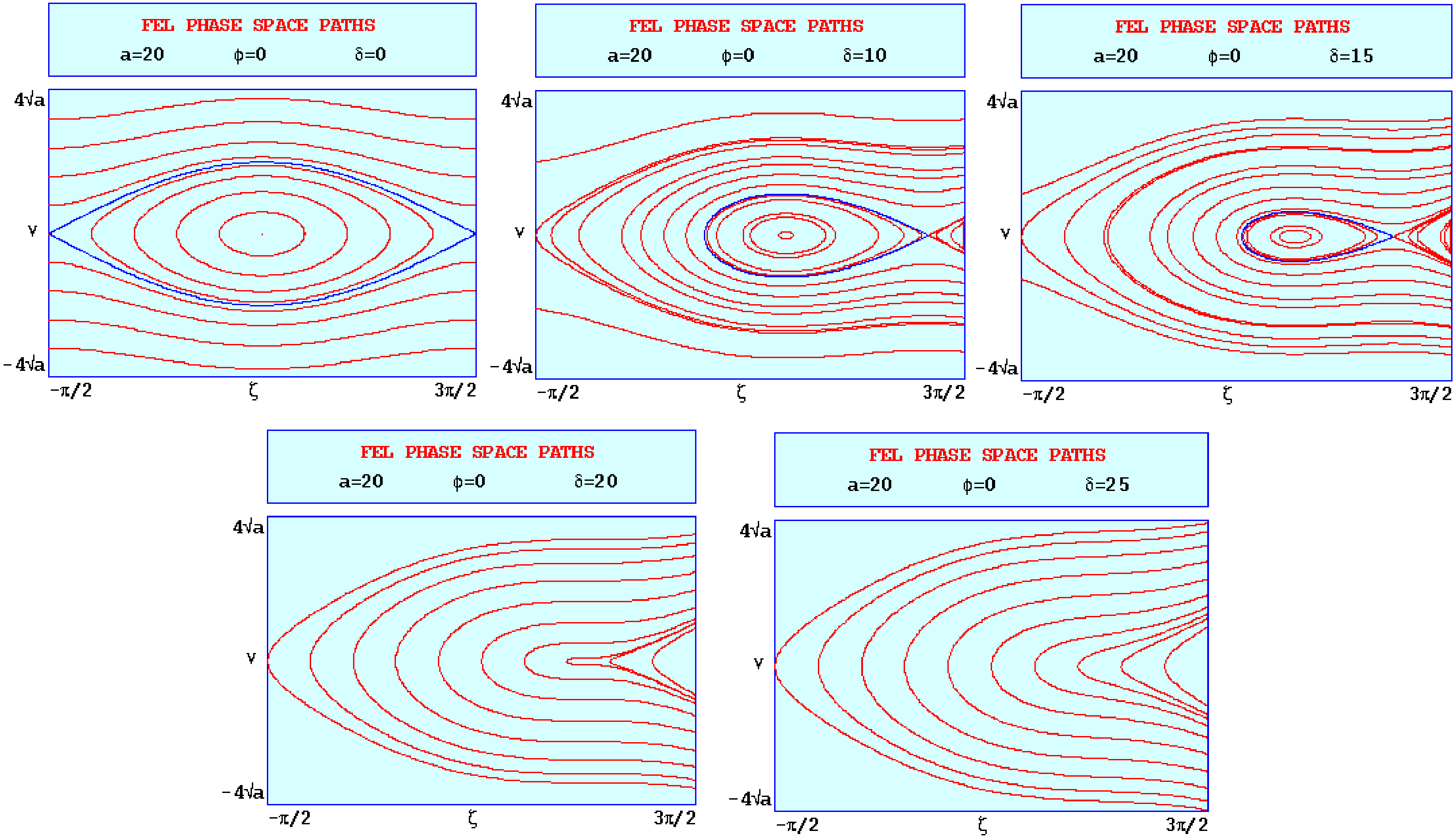
ositive Step-Taper Undulator: $\Delta > 0$

Negative Step-Taper Undulator: $\Delta < 0$

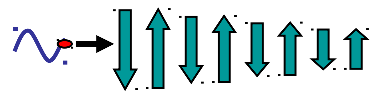


Tapered Phase Space ($\delta>0$)

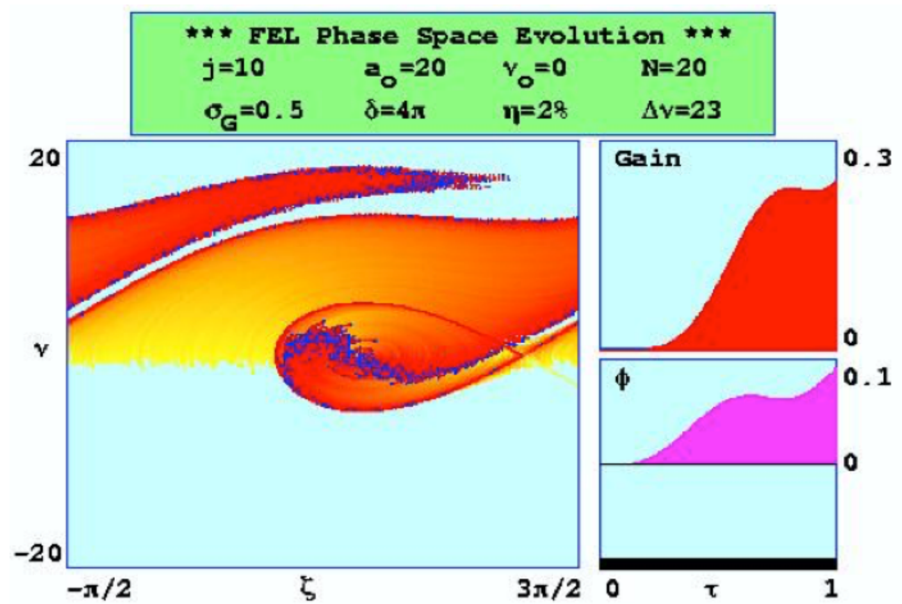
- Strong laser field $|a|=20$, increase taper δ = 0, 10, 15, 20, 25
- Closed orbit region decreases, more open orbits



Linearly Tapered Undulator: $\delta=4\pi$



- "conventional taper" $\delta=4\pi$
- electrons trapped and bunched near $\nu = 0$
- Note new separatrix due to phase acceleration δ
- extraction for $\delta=4\pi$ is



$$\boxed{\ddot{\zeta} = \dot{\nu} = \delta + |a|\cos(\zeta+\phi)}$$

$$\delta = -4\pi N \frac{K^2}{1+K^2}\frac{\Delta K}{K}$$